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RESEARCH ARTICLE

A Detector of an Invisible Carpet Cloaking Structure

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Based on transformation optics, we propose a device to detect the presence of a carpet cloaking structure at near field. By using complementary media, the detective device can optically cancel a certain volume of reflective metal and reveal the hidden medium inside the metal. The detective performance of this device is confirmed by full-wave finite-element simulations. It is shown that the detective device is invisible when there is no carpet cloak in the system, yet it becomes visible with the presence of the carpet cloak. The device works at near field, but the response can be found even at the far field. Furthermore, it is shown that the device can detect both anisotropic and isotropic carpet cloaking structures. The investigation may provide a unique method to detect a carpet cloak and contribute to the design of novel optical devices, such as far field detectors for a nanoscale medium.

Keywords: Optical Cloaking, Transformation Optics, Complementary Media, Metamaterials.

1. INTRODUCTION

Recently, the concept of transformation optics[1–2] has enabled us to design novel materials that can steer light along arbitrary curves. One of the most fascinating designs is a cloaking device which can bend light around a concealed region.[1–6] Inspired by the theoretical strategies, metamaterial microwave cloak has been experimentally realized.[7] However, some problems remain challenging, such as singular parameter and narrow-band limit of the cloak. In order to solve parameter singularity of the cloak, carpet cloaking has been proposed to give all cloaked objects the appearance of a flat conducting sheet,[8] which has been experimentally demonstrated at both microscopic[9] and macroscopic[10–11] aspects, respectively. As we know, the early approaches tended to hide an object inside the cloaked domain. Recently Lai et al.[12] proposed a different approach that allows to cloak the object at a distance outside the cloaking shell based on complementary media,[13–16] which further leads to illusion optics.[17] Up to now, the concept of cloak has been extended from electromagnetic wave to both acoustic wave[18–19] and matter wave.[20]

In contrast to the invisibility, the issue which presents how to detect an invisible cloaking has also been explored in previous studies.[21–23] For example, Chen et al.[21] firstly proposed a kind of transformation media called "anti-cloak" which locates inside the cloak shell and makes the hidden objects detectable. And a detection scheme has also been developed by shooting a fast-moving charged particle through the cloak.[22] Very recently, Zhang et al. have discussed the defect of the isotropic ground-plane carpet cloak, which may generally lead to a tiny lateral shift of the scattered wave and lead the carpet detectable.[23] Moreover, Zhao et al. proposed a hiding-inside and watching-outside carpet cloak which allows visibility outside by hiding a sensor under the carpet cloak without being detected form the outside, i.e., "seeing without being seen.[24]"

In this work, we propose a unique scheme for detecting the presence of a carpet cloak based on transformation optics. By using complementary media, the detective device can optically cancel a certain volume of reflective metal and reveals the "hidden" medium inside it. According to full-wave finite-element simulations, we find that the detective device is invisible when there is no carpet cloak in the system; while it turns to be visible with the presence of the carpet. The device works at near field, but the response can be found even at the far field. Furthermore, the device can detect both anisotropic and isotropic carpet cloaking structures.

2. THEORETICAL ANALYSIS

We start from the carpet cloak.[8] For simplicity, we consider the two-dimensional (2D) wave problem with transverse electric polarization. The carpet lies on a flat ground plane which is a highly reflective metal of $\varepsilon_1 = -1$ and $\mu_1 = 1$. The carpet cloak is composed of two parts. As shown in Figure 1(a), one is the transformation media, which mimics an empty space; the other is a highly reflective metal layer under the carpet cloak, which mimics the flat ground

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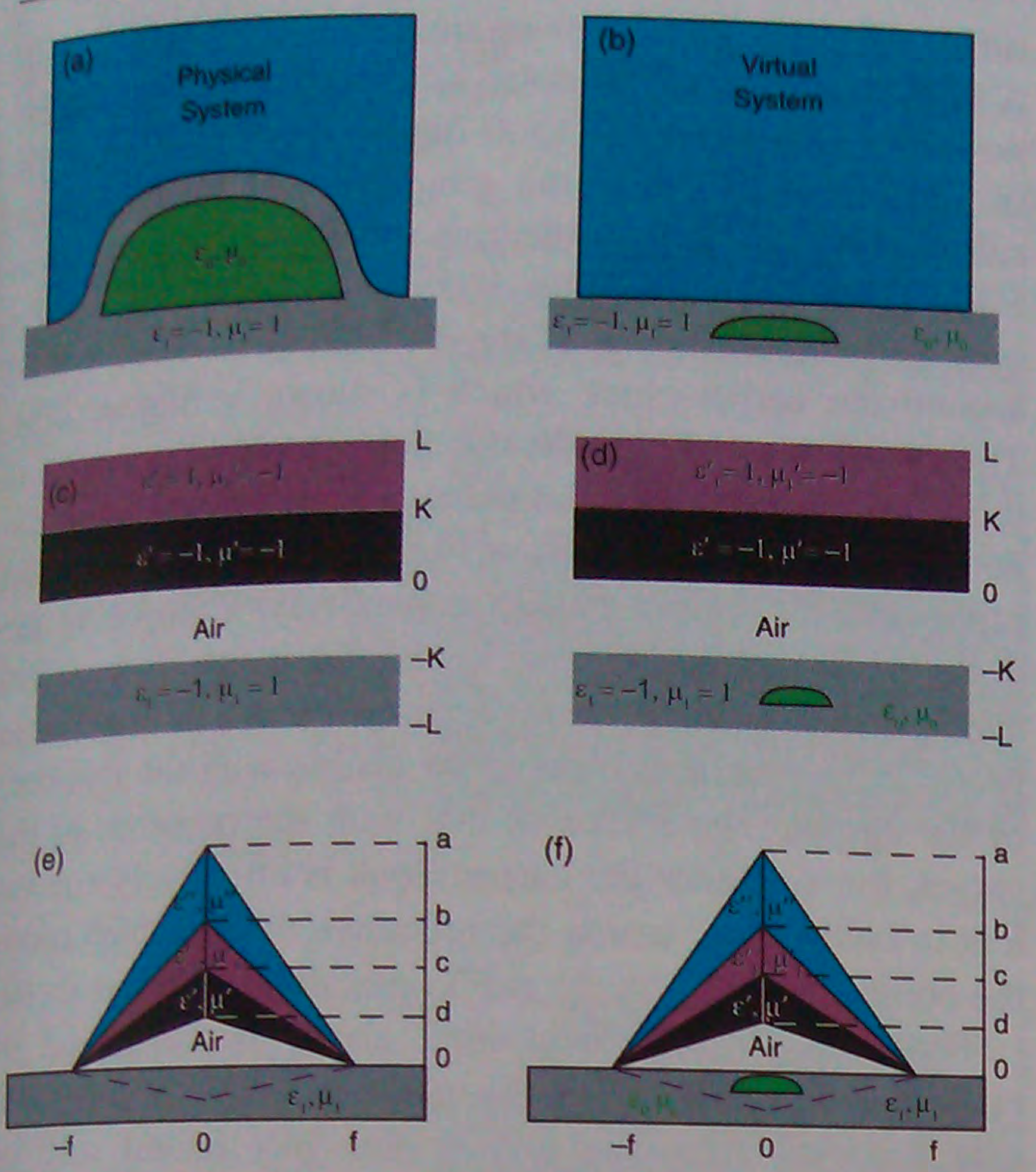

Fig. 1. The physical system (a) of a carpet cloak and the virtual system (b). The Physical system is perceived as the medium (with permittivity ε_0 and permeability μ_0) being "put" inside the flat ground plane in the virtual system. (c) The schematic to show the complementary media slabs shown in purple and black in the region of $0 < y' < L$, which optically cancel the slabs shown in white and grey in the region of $-L < y < 0$, respectively. (d) The schematic to detect the "hidden" medium because the complementary media slabs can not optically cancel the slabs of $-L < y < 0$. (e) The schematic of a finite-volume detective device consists of three couples with permittivity and permeability ε', μ' (shown in black), ε'_1, μ'_1 (shown in purple) and ε'', μ'' (shown in blue), respectively. Here, a, b, c and d are the distances from the acme of each triangles to the ground plane $y = 0$, f is the half width of the device. (f) The schematic to detect the "hidden" medium by the detective device as shown in (e).

plane. The system is perceived as a flat ground plane and the medium with ε_0 and μ_0 under the carpet is effectively equivalent to being "put" in the flat ground plane as shown in Figures 1(a)–(b). There exists difference between a flat plane and a carpet cloak on it. Consequently, we can introduce the complementary media to detect the difference.

According to transformation optics,[25] when a space is transformed into another space of different shape, the permittivity ($\boldsymbol{\varepsilon}'$) and the permeability ($\boldsymbol{\mu}'$) in the transformed space (x') are described as

$$\varepsilon^{i'j'} = \frac{1}{\det(\Lambda_i^{i'})} \Lambda_i^{i'} \Lambda_j^{j'} \varepsilon^{ij} \tag{1}$$

and

$$\mu^{i'j'} = \frac{1}{\det(\Lambda_i^{i'})} \Lambda_i^{i'} \Lambda_j^{j'} \mu^{ij} \tag{2}$$

respectively. Here ε^{ij} and μ^{ij} are the permittivity and the permeability in the original space ($\boldsymbol{x}$) and $\Lambda_i^{i'} = (\partial x^{i'}/\partial x^i)$ is the Jacobian tensor of the coordinate transformation. The complementary media can be regarded as a special kind of transformation media, which is formed by a specific coordinate transformation, i.e., folding a piece of space into another. As shown in Figure 1(c), the grey region of $y < -K$ is an infinitely large reflective metal of $\varepsilon_1 = -1$ and $\mu_1 = 1$. We take a coordinate transformation of $y' = -y$ for $-L < y < 0$, which corresponds to folding the air slab of $-K < y < 0$ into the slab of $0 < y' < K$, and folding the metal slab of $-L < y < -K$ into the slab of $K < y' < L$, respectively (Here L and K is the distance from $y = 0$ to each boundary of the two slabs). According to Eqs. (1)–(2), the black slab of $0 < y' < K$ becomes a perfect lens[13] of $\varepsilon' = -1$ and $\mu' = -1$; and the purple slab of $K < y' < L$ becomes complementary media of $\varepsilon'_1 = 1$ and $\mu'_1 = -1$. In this case, the optical phase at $y = -L$ and $y' = L$ are exactly the same, and the region $(-L, L)$ appears to be nonexistent optically. So the electromagnetic wave will be highly reflected on the upper boundary of the purple region, as it is reflected by the metal of $y < -L$. Now the medium with permittivity ε_0 and permeability μ_0 is "put" inside the metal of $-L < y < -K$ (as shown in Fig. 1(d)). The case is effectively equivalent to that the carpet cloak locates in the air slab. Therefore, the complementary media in the region of $0 < y' < L$ can not optically cancel the slabs in the region of $-L < y < 0$. After "putting" the medium inside the metal slab, the reflection of light wave of the system is different. As a result, the medium inside the metal slab can be perceived. The black and purple regions consist of complementary media can be considered as a detective device of a carpet cloak.

Now we construct a detective device with a finite volume. In Figure 1(e), we show a wedge-shaped detective device consists of complementary media. The grey region in $y < 0$ indicates a infinitely large reflective metal of $\varepsilon_1 = -1$ and $\mu_1 = 1$. The black region with parameters $\boldsymbol{\varepsilon}'$ and $\boldsymbol{\mu}'$ optically cancels the triangular region of air and the purple region with parameters $\boldsymbol{\varepsilon}'_1$ and $\boldsymbol{\mu}'_1$ optically cancels the triangle region of metal marked by purple dashed line. The black and purple regions can be obtained by an identical coordinate transformation

$$y' = -\frac{c-d}{d} y \pm \frac{c}{f} x + c \tag{3}$$

Here, a, b, c and d are the distance from the acme of each triangle to the ground plane $y = 0$, f is the half width of the device. In the Eq. (3), the signs "+" and "−" correspond to the left and right half triangular part, respectively. According to Eqs. (1)–(2), the parameters $\boldsymbol{\varepsilon}'$, $\boldsymbol{\mu}'$, $\boldsymbol{\varepsilon}'_1$ and $\boldsymbol{\mu}'_1$ can be expressed as

$$\boldsymbol{\varepsilon}' = \boldsymbol{\mu}' = \begin{pmatrix} \dfrac{-d}{c-d} & \pm\dfrac{cd}{f(c-d)} & 0 \\ \pm\dfrac{cd}{f(c-d)} & \dfrac{-d}{c-d}\left(\dfrac{(c-d)^2}{d^2} + \dfrac{c^2}{f^2}\right) & 0 \\ 0 & 0 & \dfrac{-d}{c-d} \end{pmatrix} \tag{4}$$

and

$$\begin{cases} \varepsilon'_1 = \varepsilon' \varepsilon_1 \\ \mu'_1 = \mu' \mu_1 \end{cases} \tag{5}$$

respectively. Due to the optical cancellation, the upper boundary of the purple region becomes a highly reflective surface. Then, in order to change the reflective surface into a flat ground plane, we introduce two triangular materials on the top of the purple region with the parameters

$$\varepsilon'' = \mu'' = \begin{pmatrix} \dfrac{a}{a-b} & \pm\dfrac{ab}{f(a-b)} & 0 \\ \pm\dfrac{ab}{f(a-b)} & \dfrac{a}{a-b}\left(\dfrac{(a-b)^2}{a^2}+\dfrac{b^2}{f^2}\right) & 0 \\ 0 & 0 & \dfrac{a}{a-b} \end{pmatrix} \tag{6}$$

The ε'', μ'' are obtained by coordinate transformation $y' = \pm(b/f)x + ((a-b)/a)y + b$. Thereafter, the light wave travels along the same optical path as that reflected by a flat ground plane. Then the whole detective device is invisible. However, when a carpet cloak exists on the flat ground plane, it is effectively equivalent to "put" a medium of ε_0 and μ_0 inside the reflective metal (as shown in Fig. 1(f)). Thereafter, the device can't optically cancel the metal. As a result, the whole system including the detective device and the carpet cloak become visible, we can detect the presence of the carpet cloak.

3. RESULT AND DISCUSSION

We carry out full-wave simulations using a finite element solver (COMSOL Multiphysics) to verify the validity of the detective device. Figure 2(a) demonstrates the specific structural parameters of the wedge-shaped detective device. The detective device is composed of three couples (each couple contains two triangles):

(i) the blue couple,
(ii) the purple couple, and
(iii) the black couple.

The corresponding parameters ε', μ', ε'_1, μ'_1, ε'' and μ'' of the materials in couples can be derived from Eqs. (4)–(6), respectively. Figure 2(b) shows the carpet to be detected. The structural parameters of the carpet is 8 μm wide and 3 μm high, and the height of the bump region is 400 nm. The color maps show the profile of n^2 (Here n is the refractive index of the material.). The carpet cloak device is designed by quasiconformal map and the material of the cloak is isotropic with n ranges from 0.87 to 1.29. A 200 nm layer of reflective metal (shown in white in the Fig. 2(b)) with parameters $\varepsilon_1 = -1$ and $\mu_1 = 1$ is positioned under the carpet cloak to mimic the reflective flat ground plane. For simplicity, we set the region under the reflective surface as air. The detective device and carpet cloak are both placed on a flat infinitely large reflective metal plane with parameters $\varepsilon_1 = -1$ and $\mu_1 = 1$. A Gaussian beam of TE polarization (E along the z direction) at a wavelength of 1500 nm is launched at an angle of 135° with respect to the ground plane. We begin the calculation from a flat reflective metal plane as shown in Figure 2(c). The reflective light wave at 45° is clearly observed. Figure 2(d) plots the distribution of electric field around the carpet cloak which is shown in Figure 2(b). The distribution of the electric field outside the carpet is the same as that from a flat plane. Thereafter, we cover the detective device (as shown in Fig. 2(a)) on the flat ground plane and the carpet cloak, respectively. Comparing the distributions of the electric fields shown in Figures 2(e) and (f), the detective device is invisible when there is no carpet cloak; while it turns to be visible with the presence of the carpet. The reason is that with the presence of the carpet, the air under the carpet cloak is effectively equivalent to being "put" inside the reflective metal, which cause the complementary media can't optically cancel the metal.

The carpet cloak mentioned above is composed of isotropic materials. It has been proved that the isotropic carpet suffers from the defect of a tiny lateral shift of the scattered wave, which leads to the carpet detectable.[2] Here, we prove that our device can also detect an anisotropic carpet. As shown in Figure 3(a), the anisotropic carpet with the same structural parameters as the isotropic carpet mentioned above follows the transfinite map

$$y'(x) = \begin{cases} y + 0.4(3-y)\cos(\pi x/4)^2/y & -2\ \mu\text{m} \le x \le 2\ \mu\text{m} \\ 0 & \text{otherwise} \end{cases}$$

The corresponding parameters ε' and μ' of the carpet can also be derived from Eqs. (1)–(2). The color maps show the profile of the square of average refractive index (n). The distribution of electric field around the anisotropic carpet is shown in Figure 3(b). The field patterns are indeed identical outside the carpet by carefully examining the field farther away, without the lateral shift of the scattered wave. Then we cover the carpet by the detective device and the distribution of electric field is shown in Figure 3(c). The reflected wave is deflected obviously and split into two different angles at around 30° and 60°, which indicates that the detective device is visible when the anisotropic carpet exists as the same as the isotropic one. At last, we shift the carpet cloak 5 μm away along x direction. The distribution of the electric field is shown in Figure 3(d). Similar to Figure 3(c), the reflected wave is also deflected and the whole system becomes visible. Therefore, the device is efficient in detecting both anisotropic and isotropic carpet cloaking structures.

Furthermore, it is interesting to observe the response at the far field when the detective device works at near field. We have also calculated the far-field patterns of the reflecting field as shown in Figure 4. The scattering fields are all normalized to the maximum field in the case of

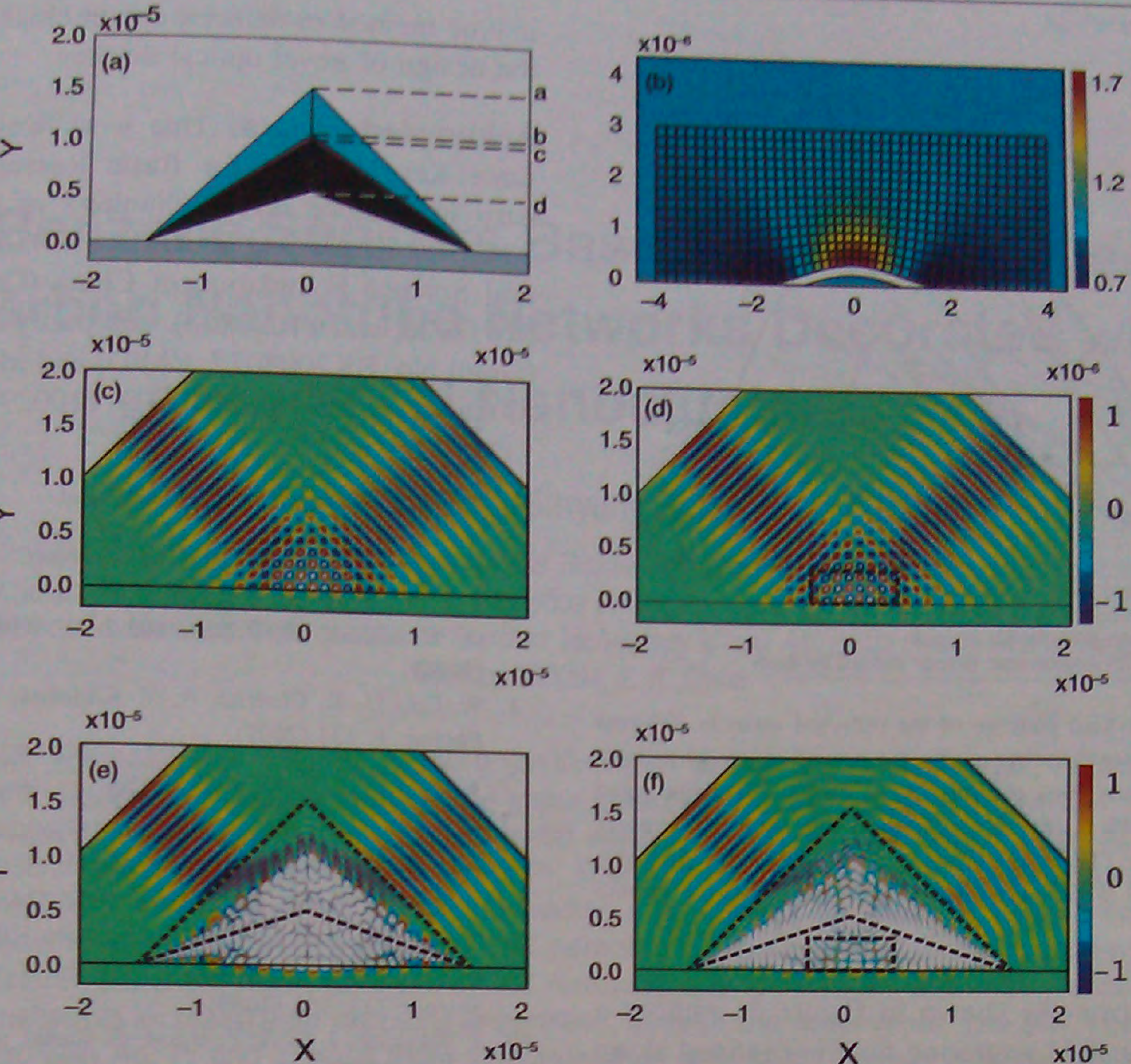

Fig. 2. (a) The geometry of the detective device in the numerical simulation. The width of the device is 30 μm and the structural parameters as $a=15$ μm, $b=10.5$ μm, $c=10$ μm and $d=5$ μm, respectively. (b) The profile of the square of refractive index (n^2) of the detected carpet shown in color map. The isotropic carpet is designed by quasiconformal map. The distributions of electric fields around: (c) a flat ground plane, (d) an isotropic carpet cloak (e) the designed detective device and (f) the carpet cloak being covered by the detective device, respectively. The dashed lines indicate the carpet and the detective device. A Gaussian beam (TE polarization, with the width as 4 μm) at a wavelength of 1500 nm is launched at 135° with respect to the ground plane.

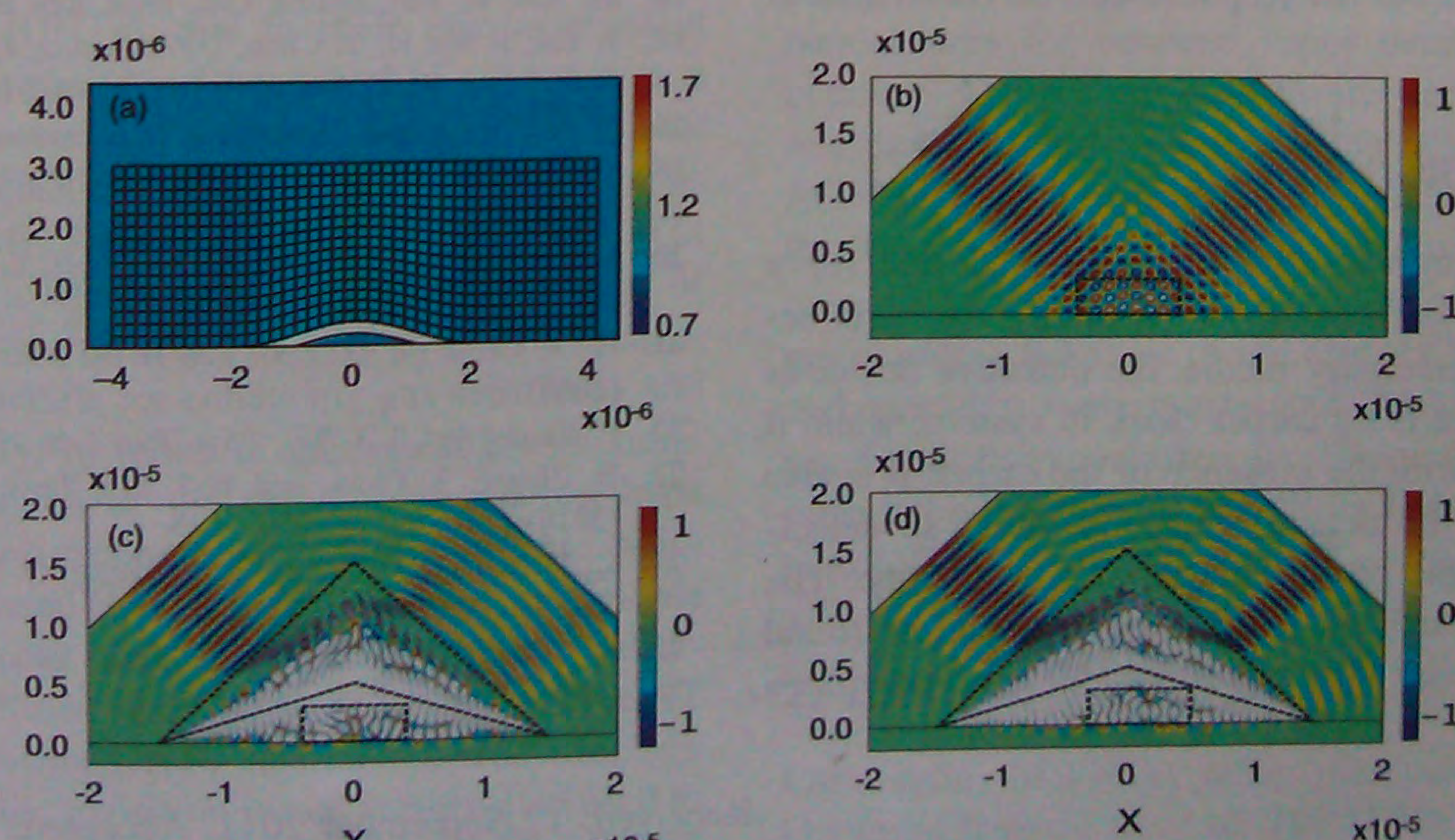

Fig. 3. (a) The profile of the square of the average refractive index (n^2) of the detected carpet shown in color map. The anisotropic carpet is designed by transfinite map. The distributions of electric field around (b) an anisotropic carpet cloak, (c) the carpet cloak covered by the detective device, and (d) the carpet with 5 μm shift along the x axis covered by the detective device, respectively. The dashed lines indicate the carpet and the detective device. A Gaussian beam (TE polarization, with the width as 4 μm) at a wavelength of 1500 nm is launched at 135° with respect to the ground plane.

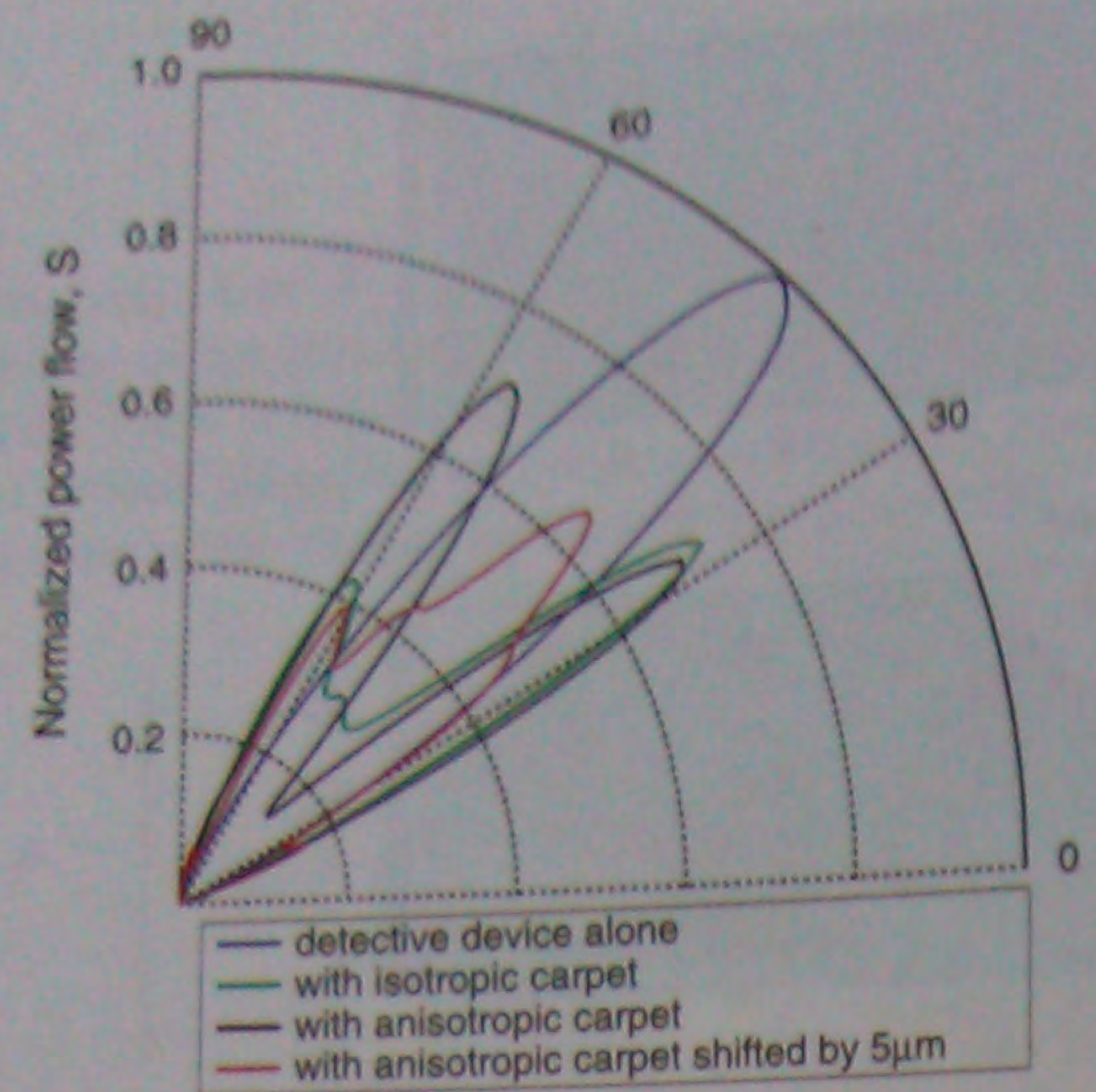

Fig. 4. Normalized far-field patterns of the reflected wave in different cases: the detective device covering on the flat ground plane (blue line), the isotropic carpet (green line), the anisotropic carpet (black line), and the anisotropic carpet with 5 μm shift along the x axis (red line), respectively. A Gaussian beam (TE polarization, with the width as 4 μm) at a wavelength of 1500 nm is launched at 135° with respect to the ground plane.

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a flat reflective mirror. As shown in Figure 4, without a carpet cloak, the far-field scattering field is confined along one specular direction ($\varphi = 45°$), indicating that the detective device is invisible at far field. While when covering the detective device on the isotropic and anisotropic carpet cloaks corresponding to the cases in Figures 2(f) and 3(c)–(d), respectively, the scattering fields split into different irregular lobes. The result makes the carpet cloaks to be visible at the far field. Obviously, the detective device works at near field, but the response can be found also at the far field.

4. CONCLUSION

In summary, we propose here a detective device for a carpet cloak based on transformation optics. By introducing definite complementary media, the detective device is invisible when there is no carpet cloak in system; while it turns to be visible with the presence of the carpet. It is also demonstrated that the device efficiently works in detecting both the isotropic carpet and the anisotropic one. The device works at near field, but the response can be found even at the far field. The investigation may provide a unique method to detect a carpet cloak and contributes to the design of novel optical devices.

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References and Notes

1. U. Leonhardt, *Science* 312, 1777 **(2006)**.
2. J. B. Pendry, D. Schurig, and D. R. Smith, *Science* 312, 1780 **(2006)**.
3. G. W. Milton, M. Briane, and J. R. Willis, *New J. Phys.* 8, 248 **(2006)**.
4. W. Cai, U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, *Nat. Photon.* 1, 224 **(2007)**.
5. U. Leonhardt and T. Tyc, *Science* 323, 110 **(2009)**.
6. M. Rahm, D. Schurig, D. A. Roberts, S. A. Cummer, D. R. Smith, and J. B. Pendry, *Photon. Nanostr. Fundam. Appl.* 6, 87 **(2008)**.
7. D. Schuring, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, *Science* 314, 977 **(2006)**.
8. J. Li and J. B. Pendry, *Phys. Rev. Lett.* 101, 203901 **(2008)**.
9. R. Liu, C. Ji, J. J. Mock, J. Y. Chin, T. J. Cui, and D. R. Smith, *Science* 323, 366 **(2009)**.
10. J. Valentine, J. Li, T. Zentgraf, G. Bartal, and X. Zhang, *Nat. Mater.* 8, 568 **(2009)**.
11. B. Zhang, Y. Luo, X. Liu, and G. Barbastathis, *Phys. Rev. Lett.* 106, 033901 **(2011)**.
12. Y. Lai, H. Chen, Z. Q. Zhang, and C. T. Chan, *Phys. Rev. Lett.* 102, 093901 **(2009)**.
13. J. B. Pendry, *Phys. Rev. Lett.* 85, 3966 **(2000)**.
14. J. B. Pendry and S. A. Ramakrishna, *J. Phys. Condens. Matter* 14, 8463 **(2002)**; 15, 6345 **(2003)**.
15. K. Kobayashi, *J. Phys. Condens. Matter* 18, 3703 **(2006)**.
16. M. Yan, W. Yan, and M. Qiu, *Phys. Rev. B* 78, 125113 **(2008)**.
17. Y. Lai, J. Ng, H. Y. Chen, D. Z. Han, J. J. Xiao, Z. Q. Zhang, and C. T. Chan, *Phys. Rev. Lett.* 102, 253902 **(2009)**.
18. S. A. Cummer and D. Schurig, *New Journal of Physics* 9, 45 **(2007)**.
19. S. Zhang, C. G. Xia, and N. Fang, *Phys. Rev. Lett.* 106, 02[illegible] **(2011)**.
20. S. Zhang, D. A. Genov, C. Sun, and X. Zhang, *Phys. Rev. Lett.* 100, 123002 **(2008)**.
21. H. Y. Chen, X. D. Luo, and H. R. Ma, *Opt. Express* 16, 14[illegible] **(2008)**.
22. B. Zhang and B.-I. Wu, *Phys. Rev. Lett.* 103, 243901 **(2009)**.
23. B. Zhang, T. Chan, and B.-I. Wu, *Phys. Rev. Lett.* 104, 2[illegible] **(2010)**.
24. J. Z. Zhao, D. L. Wang, R. W. Peng, Q. Hu, and M. Wang, *Phys. Rev. E.* 84, 046607 **(2011)**.
25. D. Schurig, J. B. Pendry, and D. R. Smith, *Opt. Express* 14, 9[illegible] **(2006)**.

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