Octopole orbital corner states in terahertz spoof plasmonic crystals with two-dimensional Su-Schrieffer-Heeger lattice

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(Received 23 August 2024; revised 30 April 2025; accepted 5 May 2025; published 20 May 2025)

Connecting topological states with p-orbital freedom has led to rich, insightful physics phenomena. Yet topological corner states have rarely been associated with modes of higher orbitals than p orbital due to the entanglement proneness of higher orbital bands in photonics. This paper theoretically investigates higher orbital corner states in a two-dimensional (2D) Su-Schrieffer-Heeger (SSH) lattice and experimentally demonstrates octopole orbital corner states in a terahertz spoof plasmonic crystal. The orbital Hamiltonian is derived by introducing a pair of orthogonal orbitals into a 2D SSH lattice tight-binding model. Based on the expanded lattice, the presence of orbital corner states is demonstrated, which can be characterized by the generalized winding number. The counterpart of the tight-binding model in plasmonic crystals is identified, revealing eight octopole corner states. These states, comprising four even and four odd modes, are demonstrated through eigenenergy spectra and field distributions. Interestingly, the octopole corner states emerge without orbital-hopping symmetry, which differs from the *p*-orbital corner states in previous reports. Experimentally, we verify the emergence of octopole orbital corner states in the terahertz frequency range by designing spoof plasmonic crystals with a 2D SSH lattice. Through terahertz time-domain spectroscopy, these corner states are directly observed via spatial mapping of the electric field intensity. We suggest that these results shed light on the unique physics of the interplay between topological phases and higher-order orbitals and broaden the potential application opportunities of plasmonic crystals in particle trapping and biosensing.

DOI: 10.1103/PhysRevB.111.195423

I. INTRODUCTION

By implementing the quantum Hall effect in classical systems [1,2], many topological schemes have been explored in different platforms [3], such as the quantum spin Hall effect or quantum valley Hall effect associated with spin [4-6] or valley [7–11] degree of freedom (DOF). In addition, the orbital properties, another fundamental feature describing the spatial distribution of wave functions inside the unit cell of crystals, unveil the great significance of exotic topological matter. The orbital bands play an essential role in correlated electronics [12] and solid-state materials [13], such as orbital superfluidity [14], topological semimetals [15], and superconductors [16]. By harnessing the orbital DOF, exotic band structures may emerge, such as a Dirac cone and flat band for the *p*-orbital bands of the honeycomb lattice [17-20] and the Lieb lattice [21]. These achievements have been facilitated by purposely engineering higher orbital bands in diverse platforms such as optical lattices of ultracold atoms [17], semiconductor polariton lattices [18,19], and artificial electronic lattices [20,21]. Moreover, the orbital study has also been extended to topological insulators. In particular, the robust orbital edge states have been verified in photonic honeycomb lattices made of coupled micropillars [22], zigzag-arranged dielectric spherical particles [23], and acoustic lattices [24,25]. Further, topological lasing has been realized based on edge modes supported by polariton micropillars [26]. Additionally, the orbital features can be used to construct pseudospin DOFs [5] and manipulate the topological phase transition [27]. These investigations expand the scope of topological phases based on orbitals.

Recently, intrinsic orbitals have been used to investigate higher-order topological insulators (HOTIs) with edge states two or more dimensions lower than the bulk [28–35]. Orbital corner states have been theoretically proposed by loading p-orbital freedom in photonic breathing kagome lattices (BKLs) [36]. Thereafter, *p*-orbital corner states have been experimentally demonstrated in BKLs of waveguide arrays [37]. In addition, a photonic quadrupole topological insulator has been achieved by leveraging both s- and p-orbital-type modes in inducing synthetic flux [38]. Meanwhile, orbital HOTIs have also been reported in acoustic systems. In particular, type-I and type-II corner states stemming from *p*-orbital interactions were theoretically proposed and experimentally observed in acoustic BKLs [39]. Based on unique *p*-orbital hopping patterns, the unusual higher-order topology was demonstrated in puckered lattice acoustic metamaterials [40]. Apart from p orbitals, higher-order orbitals have also

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attracted much attention. For instance, plasmonic higher-order topological states with orthogonal nondegenerate orbital-like hexapole modes were experimentally demonstrated based on BKLs [41]. Generally, introducing orbital DOFs can significantly enrich the coupling configurations within lattice systems, offering unique avenues for exploring topological physics.

Up to now, most research on orbital-related topological phases has been associated with p-orbital modes, and experimental realization of orbital corner states has been implemented in BKLs, where the coupling of orbital hopping has been unveiled. As a comparison, the orbital corner states in a two-dimensional (2D) Su-Schrieffer-Heeger (SSH) lattice have been theoretically proposed [42]. However, the coupling between two types of *p*-orbital hopping was not considered, where the band structure could be decoupled into two sets of s bands. Corresponding to s bands [29,30], the higher-order topological states in a 2D SSH lattice have been extensively studied, yet the orbital corner states related to higher orbital bands were overlooked when the couplings of orbitals hopping existed in the system, although the *p*-orbital corner states were demonstrated based on the dipole mode in the plasmonic crystals with BKLs [36]. Actually, the multipolar plasmonic modes with anisotropic field distributions appear at higher frequencies, and usually have sharper resonances than the dipole mode of localized surface plasmons, and hence provide us with potential opportunities of applications in the design of antennas [43,44], sensors [45-47], and nanolasers [48]. Motivated by these considerations, we may explore the higher orbital corner states based on the octopole mode in plasmonic crystals.

This work theoretically investigates higher orbital HOTIs in the 2D SSH lattice and experimentally demonstrates octopole topological corner states in terahertz spoof plasmonic crystals. Based on the tight-binding model, we derive the higher orbital Hamiltonian, where the coupling between two types of orbital hopping has been included by introducing a pair of orthogonal orbital bases set in a 2D SSH lattice. By using expanding lattices, orbital corner states depending on the orbital coupling strength are obtained. Subsequently, the generalized winding number is calculated to unveil the nontrivial topological property of orbital corner states in a 2D SSH lattice. To visualize orbital corner states, the plasmonic crystals are analogous to the tight-binding model by calculating the eigenmodes of a single cavity and coupling configurations of two cavities. The eigenfrequency spectrum and field distributions are simulated by introducing the orbital coupling strength proposed in the tight-binding model, and octopole corner states are demonstrated in a truncated square-shaped 2D SSH plasmonic crystal. Experimentally, we design and fabricate the spoof plasmonic crystals, and directly observe the octopole corner states in the terahertz regime by mapping the electric field intensity. Our approach can be extended to other systems and applied to other orders of orbital corner states. We suggest that these investigations shed light on the unique physics of the interplay between higher-order topological phases and orbital modes.

This paper is organized as follows. After the introduction, in Sec. II, the higher orbital Hamiltonian based on a 2D SSH lattice tight-binding model is derived, and orbital corner states are demonstrated by calculating the eigenenergy spectra of a finite structure. Thereafter, in Sec. III, the topological invariant characterizing orbital corner states is investigated. In Sec. IV, the counterpart for the tight-binding model in a photonic system is identified, and the octopole corner states are demonstrated based on eigenenergy spectra and field distributions in a truncated square-shaped 2D SSH plasmonic crystal. In Sec. V, a spoof plasmonic crystal with a 2D SSH lattice is designed to establish octopole corner states in the terahertz frequency. Then, the terahertz experiments are implemented to confirm the existence of octopole corner states in the fabricated finite structure of the spoof plasmonic crystal. The results are summarized in Sec. VI.

II. HIGHER ORBITAL CORNER STATES BASED ON TIGHT-BINDING MODELS

We consider a 2D SSH lattice as shown in Fig. 1(a), and the unit cell consists of four nodes denoted by $\{1, 2, 3, 4\}$. The solid and dashed lines in Fig. 1(a) stand for the intracell (t_1) and intercell (t_2) hopping, respectively. Generally, when the hopping strength $t_1 < t_2$, the 2D SSH lattice falls into a nontrivial topological phase, which supports robust corner states reported previously in single orbital HOTIs [29,30]. Here, we introduce orbital DOFs to this typical model. For a single lattice site, p, d, f, and g orbitals, as depicted in Fig. 1(b), occur in pairs and are orthogonal in the same order. As a result, interactions between two neighboring sites are analogous to the bonding and antibonding states of two atoms in condensed matter. Here, we take the g orbital as an example. There arise four coupling configurations: σ -like bonding, π -like bonding, π -like antibonding, and σ -like antibonding, as illustrated in Fig. 1(c).

By introducing a pair of orthogonal orbital modes within each site in a 2D SSH lattice, and considering the nearestneighbor couplings, the higher orbital bulk Hamiltonian in real space can be expressed as

$$H = \sum_{r} \left[t_{\sigma 1} (b_{r,1}^{\dagger} a_{r,1} + c_{r,2}^{\dagger} b_{r,2} + d_{r,3}^{\dagger} c_{r,3} + a_{r,4}^{\dagger} d_{r,4}) + t_{\sigma 2} (b_{r-e_{1},1}^{\dagger} a_{r,1} + c_{r-e_{2},2}^{\dagger} b_{r,2} + d_{r-e_{3},3}^{\dagger} c_{r,3} + a_{r-e_{4},4}^{\dagger} d_{r,4}) + t_{\pi 2} (\tilde{b}_{r-e_{1},1}^{\dagger} \tilde{a}_{r,1} + \tilde{c}_{r-e_{2},2}^{\dagger} \tilde{b}_{r,2} + \tilde{d}_{r-e_{3},3}^{\dagger} \tilde{c}_{r,3} + \tilde{a}_{r-e_{4},4}^{\dagger} \tilde{d}_{r,4}) + t_{\pi 1} (\tilde{b}_{r,1}^{\dagger} \tilde{a}_{r,1} + \tilde{c}_{r,2}^{\dagger} \tilde{b}_{r,2} + \tilde{d}_{r,3}^{\dagger} \tilde{c}_{r,3} + \tilde{a}_{r,4}^{\dagger} \tilde{d}_{r,4}) \right] + \text{H.c.},$$

$$(1)$$

where $t_{\sigma 1}$ ($t_{\sigma 2}$) and $t_{\pi 1}$ ($t_{\pi 2}$) correspond to the intracell (intercell) σ -like and π -like coupling strengths, respectively. a_r (\tilde{a}_r), b_r (\tilde{b}_r), c_r (\tilde{c}_r), and d_r (\tilde{d}_r) denote the σ -like (π -like) annihilation operators at the sites 1, 2, 3, and 4 in a unit cell located at position r, respectively. The subscripts 1, 2, 3, and 4 in Eq. (1) indicate the projection direction of the operator along the lattice vectors e_i (i = 1, 2, 3, and 4), as shown in Fig. 1(a). In order to obtain the band structure, we take the Fourier transformation for the real basis of the Hamiltonian in Eq. (1). By harnessing the eight-component spinor, $\psi = [a_{\mathbf{k},x}, a_{\mathbf{k},y}, b_{\mathbf{k},x}, b_{\mathbf{k},y}, c_{\mathbf{k},x}, c_{\mathbf{k},y}, d_{\mathbf{k},x}, d_{\mathbf{k},y}]^T$, the Hamiltonian can be described by $H = \sum_{\mathbf{k}} \psi^{\dagger} H(\mathbf{k}) \psi$. More



FIG. 1. (a) The schematic diagram for the tight-binding model. The red line represents a unit cell. The coupling strength is denoted as t_1 and t_2 , respectively. (b) Illustrations of 2D orbitals for *p*, *d*, *f*, and *g* orbitals, respectively. (c) The schematic diagram of the 2D orbital coupling for *g* orbitals. Band structures of the higher orbital model with the hopping parameters (d) $|t_{\pi 1}/t_{\sigma 1}| = |t_{\pi 2}/t_{\sigma 2}| = 0.1, t_{\sigma 1}/t_{\sigma 2} = 0.05$; (e) $|t_{\pi 1}/t_{\sigma 1}| = |t_{\pi 2}/t_{\sigma 2}| = 0.91, t_{\sigma 1}/t_{\sigma 2} = 0.01$; and (f) $|t_{\pi 1}/t_{\sigma 1}| = 0.81, |t_{\pi 2}/t_{\sigma 2}| = 0.91, t_{\sigma 1}/t_{\sigma 2} = 0.01$. The first Brillouin zone of a square lattice is shown in the inset in (d). The right panels in (e) and (f) show the enlarged views of the energy bands in the dashed boxes, respectively.

specifically, the matrix $H(\mathbf{k})$ takes the form of

$$H(\mathbf{k}) = -\begin{bmatrix} 0 & D_1^{\dagger} & 0 & D_4 \\ D_1 & 0 & D_2^{\dagger} & 0 \\ 0 & D_2 & 0 & D_3^{\dagger} \\ D_4^{\dagger} & 0 & D_3 & 0 \end{bmatrix},$$
(2)

where the symbol † denotes the Hermitian conjugate, and D_1 , D_2 , D_3 , and D_4 represent $D_1 = D_3 = \begin{bmatrix} f_{\pi 1} & 0 \\ 0 & f_{\sigma 1} \end{bmatrix}$, $D_2 = D_4 = \begin{bmatrix} f_{\sigma 2} & 0 \\ 0 & f_{\pi 2} \end{bmatrix}$ with $f_{\sigma i} = t_{\sigma 1} + t_{\sigma 2}e^{i\mathbf{k}\cdot\mathbf{e}_i}$ and $f_{\pi i} = t_{\pi 1} + t_{\pi 2}e^{i\mathbf{k}\cdot\mathbf{e}_i}$; \mathbf{e}_i (i = 1, 2) denotes the unit vectors along the hopping direction between the nearest neighboring sites. Remarkably, the coupling between two types of orbital hopping is included in the orbital Hamiltonian (2), since it cannot be decomposed into two independent Hamiltonians associated with orthogonal orbital hopping. By diagonalizing the Hamiltonian matrix in Eq. (2), we achieve the band structures with different orbital coupling strengths. We choose the hopping amplitude $t_{\sigma 2}$ as the energy unit of the model, i.e., setting $t_{\sigma 2} = -1$ hereafter in this paper. As shown in Figs. 1(d)–1(f), for each case, the band structure demonstrates eight distinct orbital bands since the Hamiltonian matrix

has been expanded into an 8×8 matrix by loading higher orbital freedom in a 2D SSH lattice. By tuning the ratio of two types of orbital coupling strength, the shape of the band structure is deformed. Notably, the orbital bands are symmetric with respect to zero energy, suggesting that the chiral symmetry is preserved in the system. Around zero energies, the band gaps are observed. Figures 1(d) and 1(e) demonstrate the band structures when the orbital coupling ratios within and between cells are identical, i.e., $|t_{\pi 1}/t_{\sigma 1}| =$ $|t_{\pi 2}/t_{\sigma 2}|$. The condition of $|t_{\pi 1}/t_{\sigma 1}| = |t_{\pi 2}/t_{\sigma 2}|$ indicates that the orbital-hopping symmetry is analogous to previous works, where *p*-orbital topological corner states have been demonstrated [36,37,39]. Besides, Fig. 1(f) shows the band structure when the orbital coupling ratios within and between cells are different, i.e., $|t_{\pi 1}/t_{\sigma 1}| \neq |t_{\pi 2}/t_{\sigma 2}|$. Meanwhile, the orbital coupling ratios are sensitive to the change of distance between two coupling sites, which may be associated with modes of higher orbitals than the p orbital. Here, we focus on the case of $|t_{\pi 1}/t_{\sigma 1}| \neq |t_{\pi 2}/t_{\sigma 2}|$ to explore the emergence of orbital corner states.

To reveal the topological phases, we further calculated the eigenenergy spectra of a truncated square-shaped orbital 2D SSH lattice possessing four unit cells on each side, with $|t_{\pi 1}/t_{\sigma 1}| = 0.81$, $|t_{\pi 2}/t_{\sigma 2}| = 0.91$, and $t_{\sigma 1}/t_{\sigma 2} = 0.01$ [as



FIG. 2. (a) The eigenenergy spectra of a finite (with 4×4 unit cells) structure based on the tight-binding model with $|t_{\pi 1}/t_{\sigma 1}| = 0.81$, $|t_{\pi 2}/t_{\sigma 2}| = 0.91$, and $t_{\sigma 1}/t_{\sigma 2} = 0.01$. The red stars represent orbital corner states, which are pinned at zero energy. (b) Energy spectra for a finite square-shaped lattice with different coupling ratios $t_{\sigma 1}/t_{\sigma 2}$ as $|t_{\pi 1}/t_{\sigma 1}| = 0.81$ and $|t_{\pi 2}/t_{\sigma 2}| = 0.91$. (c) Energy spectra for a finite square-shaped lattice with different coupling ratios $t_{\pi 1}/t_{\sigma 1}$ as $|t_{\pi 2}/t_{\sigma 2}| = 0.91$ and $t_{\sigma 1}/t_{\sigma 2} = 0.01$. (d) Energy spectra for a finite square-shaped lattice with different coupling ratios $t_{\pi 2}/t_{\sigma 2}$ as $|t_{\pi 1}/t_{\sigma 1}| = 0.81$ and $t_{\sigma 1}/t_{\sigma 2} = 0.01$. (d) Energy spectra for a finite square-shaped lattice with different coupling ratios $t_{\pi 2}/t_{\sigma 2}$ as $|t_{\pi 1}/t_{\sigma 1}| = 0.81$ and $t_{\sigma 1}/t_{\sigma 2} = 0.01$.

shown in Fig. 2(a)]. It follows that there are eight degenerated corner states located at zero energy between a band gap. Further, the impact of different coupling parameters on the orbital corner states is investigated in Figs. 2(b)-2(d). The energy spectra are calculated as a function of $t_{\sigma 1}/t_{\sigma 2}$ in the conditions $|t_{\pi 1}/t_{\sigma 1}| = 0.81$ and $|t_{\pi 2}/t_{\sigma 2}| = 0.91$, as shown in Fig. 2(b). The higher orbital topological corner states denoted by the red line arise for small $t_{\sigma 1}/t_{\sigma 2}$ ratios. To test the mutual interactions of orbital hopping, the energy spectra are calculated as a function of the orbital interaction $t_{\pi 1}/t_{\sigma 1}$ and $t_{\pi 2}/t_{\sigma 2}$, respectively, as shown in Figs. 2(c) and 2(d), where the intracell and intercell hopping ratios are kept constant with $t_{\sigma 1}/t_{\sigma 2} = 0.01$. It follows that the orbital interactions between the intercell have a greater effect on the generation of orbital corner states than those of the intracell. Consequently, the emergence of orbital corner states depends not only on the relative coupling strength between intracell and intercell but also on the relative orbital coupling strength.

III. ANALYSIS OF TOPOLOGICAL INVARIANT BASED ON HIGHER ORBITAL HAMILTONIAN

In order to confirm that the system based on higher orbital modes is indeed a topologically protected HOTI, the generalized winding number is employed to identify the topological properties of the higher orbital corner states. The expanded square lattice can be regarded as the generalization of the 1D SSH model, and it can be divided into four subgroups with the components of the Hamiltonian matrix due to C_4 symmetry. The generalized winding number can be defined as [37,49]

$$\mathcal{W}_i = \frac{1}{2\pi} \int_0^{2\pi} dk_i \frac{d\Phi_i(k_i)}{dk_i},\tag{3}$$

where k_i is the wave vector along the direction e_i (i = 1, 2, 3, 4) and varies across the Brillouin zone. $\Phi_i(k_i)$ is the argument of det $[H'_i(\mathbf{k})]$, and $H'_i(\mathbf{k})$ is the matrix elements of $H'(\mathbf{k})$ along the e_i direction. The winding number along the k_i direction describes the whole system, where we have $W_1 = W_2 = W_3 = W_4$, as the C_4 symmetry is maintained in the system. Here, the auxiliary Hamiltonian $H'(\mathbf{k})$ is introduced, which is derived from a unitary transformation of $H(\mathbf{k})$. The transform matrix operator is defined as

$$R = \begin{bmatrix} r_1 & 0 & 0 & 0\\ 0 & r_2 & 0 & 0\\ 0 & 0 & r_3 & 0\\ 0 & 0 & 0 & r_4 \end{bmatrix},$$
(4)

where $r_i = r(\theta_i) = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{bmatrix}$ $(i = 1, 2, 3, 4), \ \theta_1 = \pi/2, \ \theta_2 = \pi, \ \theta_3 = 3\pi/2, \ \text{and} \ \theta_4 = 0, \ \text{respectively. Then, the}$ auxiliary Hamiltonian can be derived as $H'(\mathbf{k}) = R^{\dagger}H(\mathbf{k})R$, and its specific form takes

$$H'(\mathbf{k}) = \begin{bmatrix} 0 & H_{1}^{\dagger} & 0 & H_{4}^{\prime} \\ H_{1}^{\prime} & 0 & H_{4}^{\prime\dagger} & 0 \\ 0 & H_{2}^{\prime} & 0 & H_{3}^{\prime\dagger} \\ H_{2}^{\prime\dagger} & 0 & H_{3}^{\prime} & 0 \end{bmatrix},$$
(5)

with $H'_i = \begin{bmatrix} t_{\sigma_1} + t_{\sigma_2} e^{ik_i} & 0\\ 0 & t_{\pi_1} + t_{\pi_2} e^{ik_i} \end{bmatrix} r(3\pi/2)$. The concrete form of generalized Pauli matrix σ_i (for k_i direction) becomes

As a result, $H'(\mathbf{k})$ can be decomposed to

$$H'(\mathbf{k}) = \sum_{i} \sigma_i \otimes H'_i + \text{H.c.}$$
(7)

Then, the determinant can be denoted as $d_i(k_i) = det[H'_i(k_i)] = (t_{\sigma 1} + t_{\sigma 2}e^{ik_i})(t_{\pi 1} + t_{\pi 2}e^{ik_i})$. When we put this complex number on the complex plane, it represents a vector. The vectors $\mathbf{d}_i(k_i)$ describing the geometrical features of the generalized winding number for the 2D SSH lattice are given by

$$\mathbf{d}_i(k_i) = (\operatorname{Re}[d_i(k_i)], \operatorname{Im}[d_i(k_i)])^T.$$
(8)

In this way, topological invariants can be obtained via the number of times that the amplitude of $\mathbf{d}_i(k_i)$ goes around the origin of the d_{ix} - d_{iy} plane as the momentum coordinate k_i sweeps across the Brillouin zone. In Fig. 3(a), we plot the



FIG. 3. (a) The curve illustrates the winding for the point of the star with the parameters $|t_{\pi 1}/t_{\sigma 1}| = 0.81$, $|t_{\pi 2}/t_{\sigma 2}| = 0.91$, and $t_{\sigma 1}/t_{\sigma 2} = 0.01$. (b) Generalized winding numbers with varying parameters. The star corresponds to this work, and the dark open dots denote the other systems [36,37,39], which are always lying on the white dashed diagonal line.

complex curves described by the end point of the $\mathbf{d}_1(k_1)$ vector in Eq. (8) with the parameters $|t_{\pi 1}/t_{\sigma 1}| = 0.81$, $|t_{\pi 2}/t_{\sigma 2}| =$ 0.91, and $t_{\sigma 1}/t_{\sigma 2} = 0.01$. Specifically, the closed curve corresponds to the point of the star in Fig. 3(b), where the number of loops of \mathbf{d}_i around the origin matches the winding-number value $\mathcal{W} = 2$.

It follows that we can summarize the phase diagram of the generalized winding number as functions of $|t_{\sigma 1}/t_{\sigma 2}|$ and $|t_{\pi 1}/t_{\pi 2}|$ as illustrated in Fig. 3(b). The generalized winding number has three values, which are W = 2 for orbital corner states, whereas W = 0 according to a topologically trivial situation. The case for W = 1 indicates that only one orbital is topologically nontrivial while the other is trivial. Intuitively, one might think that half of the corner states should persist. However, as the trivial and nontrivial subspaces are coupled, the corner states of the whole system merge into the bulk state [37], and no corner states exist within the band gap. Moreover, the value of $t_{\sigma 1}/t_{\sigma 2}$ plays an essential role in determining the



FIG. 4. (a) Schematic diagram for the photonic square lattice. The white circles represent cavities with radii r_0 . d_1 and d_2 represent the distances between the nearest cavities. Here, the radius of each air cavity is fixed at $r_0 = 100$ nm in the related calculations. (b), (i) shows the calculated eigenfrequencies for a single air cavity (the inset). (ii) shows the field distribution E_z for partial eigenmodes, which are p-, d-, f-, and g-orbital modes from the left to right columns, respectively. (c) The calculated spatial field distributions of four distinct eigenmodes in the two coupled cavities for g orbitals, where the distance of two cavities is set as d = 300 nm. (d) $|t_{\pi}/t_{\sigma}|$ as a function of the coupling distance d between two cavities for g orbital. (e) The spectrum of eigenfrequency deviation $\delta f = f - f_0$, where $f_0 = 3155.384$ THz, and f denotes the eigenfrequency of the truncated plasmonic crystal with 4×4 unit cells. Here $d_1 = 350$ nm and $d_2 = 250$ nm. (f) Electric field in the quarter area of the 4×4 unit cells corresponding to the eight orbital corner states in (e) with four even modes (the top row) and four odd modes (the bottom row). The right top and bottom panels show one of the enlarged corner states in the top and bottom rows, respectively. In the calculations, the unit of electric field E_z is V/m.

existence of the orbital corner states. This coincides with the condition for the emergence of a single orbital corner state.

IV. OCTOPOLE CORNER STATES IN PLASMONIC CRYSTALS

Subsequently, we identify the photonic counterparts of our higher orbital model for implementation in a photonic system. The designed photonic system consists of a 2D SSH lattice of circular air cavities embedded in a metallic background, as depicted in Fig. 4(a), and the details are presented in the inset of the right panel. The permittivity of background materials is expressed as a Drude metal, $\varepsilon = 1 - \omega_p^2 / (\omega^2 + i\gamma\omega)$, where the plasmon frequency of aluminum $\omega_p = 2.29 \times 10^{16} \text{ s}^{-1}$ is adopted, and the Drude loss γ is ignored for the sake of simplicity. The calculations are carried out with a commercial software package (COMSOL MULTIPHYSICS), and transverse magnetic polarizations are considered. In Fig. 4(b), the calculated eigenfrequencies for a single air cavity (schematically shown in the inset) are demonstrated in (i). The calculated electric fields E_z of some eigenmodes for a single air cavity are shown in (ii) of Fig. 4(b). It is demonstrated that an air cavity hosts multiple eigenmodes, such as dipole, quadrupole, hexapole, and octopole modes, just as p, d, f, and g orbitals, respectively, of atoms from quantum mechanics. For each order of orbitals, a pair of degenerate orthogonal modes exists [as shown in each column of (ii) in Fig. 4(b)].

Two coupled cavities may result in symmetry and antisymmetry modes. For g-orbital eigenmodes, there are four coupling configurations: σ -like bonding, π -like bonding, π like antibonding, and σ -like antibonding, which correspond to the increasing resonant frequencies ω_{σ}^{g} , ω_{π}^{g} , $\omega_{anti-\pi}^{g}$, and $\omega_{anti-\sigma}^g$, respectively [as shown in Fig. 4(c)]. Since the field distribution of each eigenmode is tightly localized within an individual air cavity, the interaction between adjacent cavities originates from the coupling of evanescent waves in the metallic host medium. We define two hopping parameters as t_{σ} and t_{π} to characterize the interplay between coupled cavities corresponding to σ -type bonding and π -type bonding coupling, respectively. According to [34] and [36], the hopping parameters are determined by $t_{\sigma} \propto (\omega_{\sigma}^g - \omega_{anti-\sigma}^g)/2$ and $t_{\pi} \propto (\omega_{\text{anti}-\pi}^g - \omega_{\pi}^g)/2$, where the hopping amplitudes between two coupled cavities are determined by half of the difference of the frequencies of the even and odd hybridized modes. Subsequently, the ratio of hopping amplitude t_{π}/t_{σ} can be obtained. Figure 4(d) shows the ratio of hopping amplitude $|t_{\pi}/t_{\sigma}| = |\omega_{\text{anti}-\pi}^g - \omega_{\pi}^g|/|\omega_{\sigma}^g - \omega_{\text{anti}-\sigma}^g|$ [34,36] against the coupling distance *d* between two cavities. It is evident that $|t_{\pi}/t_{\sigma}|$ decreases linearly with the distance d. Further, the range of $|t_{\pi}/t_{\sigma}|$ is from 0.91 to 0.81 when d varies from 250 nm to 350 nm.

To visualize the higher orbital topological corner states, we further show the simulated results of finite square-shaped 2D SSH photonic structures with 4 × 4 unit cells. The intracell distance d_1 and intercell distance d_2 are set as 350 nm and 250 nm, which correspond to $|t_{\pi 1}/t_{\sigma 1}| = 0.81$ and $|t_{\pi 2}/t_{\sigma 2}| = 0.91$, respectively. Figure 4(e) shows the spectrum of eigenfrequency deviation $\delta f = f - f_0$, where $f_0 = 3155.384$ THz, and f denotes the eigenfrequency of the truncated

plasmonic crystal with 4×4 unit cells. Due to the existence of electromagnetic long-range interactions, the eigenfrequency spectrum is not completely consistent with the tight-binding model. However, there are obviously eight corner states in the middle bulk band gap. Accordingly, the electric field distributions E_7 of the eight corner states are presented in Fig. 4(f). The corner states do exhibit octopole orbital configurations, which can be categorized into two groups: four even modes distributed in four corners of the finite structure [shown on the top panel in Fig. 4(f) and four odd modes scattered in four corners of the finite structure [shown at the bottom panel in Fig. 4(f)]. Thus, eight octopole corner states with four odd and four even modes have been identified in 2D SSH plasmonic crystals using the orbital coupling strengths proposed in the theoretical analysis corresponding to the point of the star in Fig. 3(b). Interestingly, the octopole corner states emerge without orbital-hopping symmetry, i.e., $|t_{\pi 1}/t_{\sigma 1}| \neq |t_{\pi 2}/t_{\sigma 2}|$. This feature is different from the *p*-orbital corner states reported in previous studies with the orbital coupling strengths marked by the dark open dots in Fig. 3(b) [36,37,39].

V. OCTOPOLE CORNER STATES IN TERAHERTZ SPOOF PLASMONIC CRYSTALS

Since the higher-order orbital bands generally correspond to the higher energy levels and metals' inherent losses in the optical frequency range [50], it is challenging to experimentally demonstrate the octopole corner states in optical frequencies. Our current work is to verify the emergence of octopole orbital corner states. Although metals are usually regarded as perfect electric conductors without electromagnetic losses in the terahertz regime, it can support surface modes by drilling an array of holes or etching subwavelength slits in the surface [51, 52]. The dispersion properties and spatial confinements of such spoof surface plasmons are similar to those of natural surface plasmons at optical frequencies. To ensure that the verification is unambiguous, the spoof plasmonic crystal constructed by textured ultrathin metal disks is employed in the experiment. In fact, a texture ultrathin metal disk supporting the localized surface plasmons has been extensively investigated as a low-frequency analogy to the plasmonic cavity in nano-optics [41,53–55]. Therefore, these disks are arranged in a spoof plasmonic crystal as an analog of a real plasmonic crystal to verify the emergence of octopole orbital corner states.

Here, we implement the 2D SSH lattice on a spoof plasmonic crystal with textured ultrathin metal disks at the terahertz regime, which is arranged with *N* periodically radical grooves on a glass substrate, as shown in Fig. 5(a). The outer and inner radii and thickness of the textured disk are *R*, *r*, and h_0 , respectively. The number of grooves is fixed as N = 24, the inner radius is fixed as r = 0.4R, and the substrate thickness is chosen as $h_1 = 0.5$ mm [as shown in the inset of Fig. 5(b)]. Due to a very small thickness ($h_0 = 50$ nm) compared with the lateral size of the disk, we take the disk array as a 2D planar structure in the calculation. Since the wave penetration depth into metal is much smaller than the wavelength, the metal is treated as a perfect electrical conductor at terahertz frequencies. For such a textured metallic disk, the calculated scattering crossing section (SCS) spec-



FIG. 5. (a) Schematics of the spoof plasmonic crystal with a square lattice, where an enlarged unit cell is presented on the right panel. D_1 and D_2 are the distances between the nearest disks, and $a = D_1 + D_2$ is the lattice constant of the spoof plasmonic crystal. (b) The left panel demonstrates the calculated SCS spectrum of a textured metallic disk placed on a dielectric substrate (schematically shown in the inset). In the calculation, the outer and inner radii are $R = 200 \,\mu\text{m}$ and r = 0.4R, respectively. The number of grooves is fixed as N = 24 with the the filling ratio of 50%. The thickness and the index of refraction of the dielectric substrate are $h_1 = 0.5 \,\text{mm}$ and n = 2.04, respectively. The arrows marked by i, ii, iii, and iv indicate resonant peaks. The right panel shows the field distribution corresponding to each peak. Obviously, octopole resonance (marked by iv) is around 0.30 THz. (c) Simulated eigenfrequency spectrum of finite spoof plasmonic crystals with 4×4 unit cells, and $D_1 = 445 \,\mu\text{m}$ and $D_2 = 415 \,\mu\text{m}$. The red open dots represent the corner states. (d) I–VIII show the electric fields E_z of the eight octopole orbital corner states in the finite spoof plasmonic crystals with 4×4 unit cells. d_1-d8 present the intensity (red open dots) around each disk as a function of its position d along the diagonal (antidiagonal) lines, with the starting point located at the center of the first disk (marked by dashed box) and ending at the center of the fourth disk, respectively. The black dashed lines are fittings. The insets in d1–d8 show the corner states inside of the black dashed boxes in I–VIII.

trum is illustrated in Fig. 5(b), which consists of multiple well-separated resonance peaks corresponding to the dipole, quadrupole, hexapole, and octopole modes from low to high frequencies. The octopole resonates around 0.30 THz, and the quality factor of the octopole mode is larger than that of the dipole, quadrupole, and hexapole modes. This feature indicates the advantage of utilizing the octopole mode for plasmonic sensing and trapping. We arrange the disks in a

square lattice with the unit cell shown as the inset of Fig. 5(a). The separations between disks are $D_1 = 445 \ \mu\text{m}$ and $D_2 = 415 \ \mu\text{m}$, respectively. The unit cell is compactly placed periodically with spatial period $a = D_1 + D_2$. Figure 5(c) shows the simulated eigenfrequency spectrum of finite spoof plasmonic crystals with 4 × 4 unit cells. There are eight octopole corner states around 0.30 THz marked by the red open dots, which are interspersed among the bulk states marked by blue



FIG. 6. (a) The photograph of the fabricated sample. The scale bar is 200 μ m. (b) Experimental setup to perform transmittance measurements. (c) Measured time-domain spectra when the terahertz waves transmit vertically through the sample; the reference spectrum is that of a bare glass substrate. (d) Measured transmission spectrum of the sample.

open dots. The simulated electric field distributions of these eight octopole corner states [as shown in Fig. 5(d)] indicate

four even and four odd modes, which distinctly manifest unique features of orbital patterns [see the insets in d1-d8 of Fig. 5(d)]. Additionally, due to the limited size of the simulated model, some nonzero electric field distributions appear inside the 4×4 unit cells, as shown in I–VIII of Fig. 5(d). Yet the corner states are exponentially localized at the corners. To illustrate the localization of the orbital corner states, we analyze the intensity $|E_{z}|^{2}$ around each disk along diagonal (or antidiagonal) lines starting from the center of the disks marked by dashed boxes in I-VIII of Fig. 5(d) and extending for 4 disks, respectively. Here, the intensity around each disk is obtained by integrating $|E_z|^2$ within an area of $430 \times 430 \,\mu\text{m}^2$ covering the disk. The integrated intensity around each disk is plotted in d1-d8 of Fig. 5(d) with red open dots, and the fittings are shown as the black dashed lines. The exponential decay of the intensity suggests the spatial localization of the corner states in the system.

Experimentally, the spoof plasmonic crystals of textured metallic disk arrays are fabricated based on the above design. The process of experimental fabrication is as follows. First, the structure of textured metallic disks is fabricated with maskless photolithography and lift-off technology. Specifically, a layer of positive photoresist film (S183) (2 μ m in thickness) is spin-coated on a 0.5 mm thick glass substrate. Then, the ultraviolet lithography is applied to generate the inverse pattern of the disks. After development, an array of



FIG. 7. (a) Experimental setup of an optical path for detecting the corner states. (b) Illustrations of spoof plasmonic crystals. The electric component of the incident wave is along the *y* axis, and the wave vector is along the *x* axis. (c) Simulated intensity distribution of the electric field under the excitation at 0.29 THz across the 5×5 unit cells. Here, the probe is located 20 µm above the sample. (d) The measured spatial distribution of electric field intensity corresponding to different regions in (b), where the yellow boxes indicate the partial sample, and the red dashed boxes highlight the position of the corner disk. (e) The images from left to right show the intensity (red open dots) around each disk as a function of its position *d* along the diagonal lines starting from corners C1, C2, C3, and C4 in regions I, II, III, and IV, respectively. The black dashed lines are the fittings of the intensity data.

inverted patterns is formed in the photoresist layer. A 50 nm thick gold layer is then deposited on the whole sample by electron beam evaporation. Thereafter, with chemical lift-off technology, those regions covered with the photoresist layer are removed, and the gold patterns in contact with the glass substrate survive. The fabricated sample size is 9.46×9.46 mm, containing 11×11 unit cells. The top-view optical micrograph of the partial sample is shown in Fig. 6(a).

The optical measurements are carried out by a terahertz time-domain spectrometer (THz-TDS, EKSPLA/THz, Lithuania), as schematically illustrated in Fig. 6(b). Photoconductive antennas are used as terahertz emitters and detectors. Excited by a femtosecond laser pulse, the antenna emits a linearly polarized terahertz beam along the y direction. We make the terahertz waves incident vertically on the sample surface, and the transmitted signals are collected by the detector. We first consider the time-domain signal of the bare glass substrate as the reference and then place the sample into the THz-TDS to obtain signal $\mathbf{E}(t)$, as shown in Fig. 6(c). After the Fourier transform, the transmission spectrum of the sample is retrieved, as shown in Fig. 6(d). There is a dip within 0.29–0.31 THz, indicating a resonance mode in this frequency range. The resonant frequency of this mode is consistent with the calculated eigenfrequency of octopole corner states in Fig. 5(c).

Then, we set up an optical system to detect the corner states, as illustrated in Fig. 7(a). We achieve a glancing incidence of terahertz waves onto the sample surface from the side by utilizing mirrors. As indicated in Fig. 7(b), the sample is placed in the x-y plane with two diagonal lines along the xand y directions, respectively. While the y-polarized terahertz wave is incident on the sample in the x axis, a 0.2-mm-width probing antenna is placed vertically in front of the sample surface to capture the radiation signals [shown in Fig. 7(a)]. To demonstrate the existence of octopole corner states, the electric field intensities $|E_z|^2$ on the sample have been simulated in advance. The simulated near-field response with a probe 20 μ m above the sample is shown in Fig. 7(c). It is distinctly evident that the octopole corner states are excited at three corners of the finite structure at 0.29 THz. Experimentally, the time-domain signals of different portions of the sample can be detected, and the spatial mapping of the electric field intensity of the sample is achieved via the Fourier transform. We conduct measurements on four corner regions I, II, III, and IV of the sample [as shown in Fig. 7(b)]. Each region has the size of 3×3 unit cells (with a side length of 2.58 mm). In Fig. 7(d), the measured patterns of regions I, II, and III are demonstrated at the frequency of 0.29 THz. Indeed, there are three localized bright spots distributed at three corners of the sample, respectively, which are consistent with the simulation in Fig. 7(c). In order to get the corner state in region IV, we rotate the sample counterclockwise by 90 degrees in the x-y plane with an incidence in the x axis. Meanwhile, the measured pattern of region IV is present in Fig. 7(d), where a bright spot at the corner can be identified. Therefore, the octopole corner states are experimentally observed at four corners marked by red dashed boxes C1, C2, C3, and C4 in Fig. 7(d), respectively, in the truncated square-shaped 2D SSH spoof plasmonic crystal. To quantify the localization of the corner states, we analyze the intensity $|E_z|^2$ around each disk along diagonal lines starting from corners of C1, C2, C3, and C4 in regions I, II, III, and IV, respectively. Here, the intensity around each disk is obtained by taking the average of $|E_z|^2$ within an area of $430 \times 430 \ \mu\text{m}^2$ covering the disk. The intensity as a function of the disk position *d* is demonstrated in Fig. 7(e) from left to right with red open dots, corresponding to regions I–IV of Fig. 7(d), respectively. The fittings are demonstrated by the black lines in Fig. 7(e), which demonstrate exponential decay and suggest the occurrence of the exponentially localized corner states. It is noted that the field confinement in region I is better than the other cases, which might come from the imperfection in fabricating the disks.

There are some inherent technical limitations and restrictions in our measurements in probe size and imaging resolution, which make it difficult to achieve the details of the profile of the octopole corner states. To identify the orbital configuration of these corner states, near-field



FIG. 8. (a) Part of the fabricated sample with a defect near the corner C1 in region I, where the disk enclosed by white dashed box is deformed. The scale bar is 400 μ m. (b) Simulated intensity distribution map corresponding to region I where the defect is introduced near the corner C1. The simulation is excited at 0.29 THz, and the probe is located 20 μ m above the sample. (c) The intensity (red open dots) at each disk as a function of its position *d* along diagonal line starting from the corner C1 in (b). (d) The measured spatial distribution of the electric field intensity in region I with a defect located near the corner C1. The yellow box indicates the partial sample. (e) The intensity (red open dots) at each disk as a function of its position *d* along the diagonal line starting from the corner C1. The yellow box indicates the partial sample. (e) The intensity (red open dots) at each disk as a function of its position *d* along the diagonal line starting from the corner C1 in (d). The black dashed lines in (c) and (e) are the fittings of intensity data.

scanning terahertz spectroscopy is required according to our calculated electric field distributions. The spatial resolution of the near-field scanning terahertz spectroscopy should possesses a resolution better than 30 μ m. The near-field detection should be able to detect the phase distribution. Also the terahertz wave should be obliquely incident on the surface of the sample. The near-field scanning terahertz spectroscopy based on a photoconductive microprobe [56] may be able to provide detailed near-field information of the octopole corner states, which is beyond our current detection capability.

Despite the limitation in detecting the near-field intensity distribution, we are able to investigate the robustness of the corner states. We introduce a defect near the corner C1 in region I, and fabricate the sample as shown in Fig. 8(a), where the disk marked by the white dashed box is deformed. We simulate the scenario where the terahertz waves illuminate the sample at 0.29 THz, and plot the near-field response, as shown in Fig. 8(b). The simulation shows that the octopole corner states still exist locally at the corner C1 of the structure, despite the existence of a defect near that corner, and the C_{4v} symmetry has been broken in the sample. Further, we measure the spatial electric field distribution of region I, as shown in Fig. 8(d). It turns out that there still exists a local bright spot at the corner C1 of the sample, which is consistent with the simulation [Fig. 8(b)]. With the same approach, we also analyze the intensity $|E_z|^2$ on each disk along the diagonal line starting from corner C1 of Figs. 8(b) and 8(d), respectively. The intensity as a function of the disk position d is demonstrated in Figs. 8(c) and 8(e) with red open dots, while the black dashed lines are the fitting results. The fitting confirms the exponential decay envelope, which is a hallmark feature of the corner states. Therefore, we conclude that the octopole corner states do exist even in the presence of defects near that corner.

VI. CONCLUSION

In summary, we theoretically demonstrate higher orbital HOTIs in the C_4 symmetric square lattice and experimentally observe the existence of octopole topological corner states in spoof plasmonic crystals. With the tight-binding model, we derive the higher orbital Hamiltonian by introducing a pair of orthogonal orbitals into the lattice. Based on the expanded square lattice, we investigate the higher energy band and identify the orbital corner states. These orbital corner states are characterized by the generalized winding number due to the C_4 symmetry of the system. Apart from tight-binding model calculations, the topological corner states of orbital-like octopole modes are also numerically simulated in the photonic system using a Drude metal with air holes. The eigenenergy spectrum and related field distributions confirm the octopole orbital characterizations of corner states, which are quite different from previous s-orbital corner states. Further, we design spoof plasmonic crystals at the terahertz frequency regime. We confirm the existence of octopole corner states at the terahertz frequency via eigenspectra and field distribution simulations. Experimentally, we identify the octopole corner states from the spatial mapping of the electric field intensity by utilizing terahertz time-domain spectra. We expect our results to promote the studies of the exotic higher orbital bands and higher-order topological physics.

ACKNOWLEDGMENTS

This work was supported by the National Key R&D Program of China (Grants No. 2022YFA1404303 and No. 2020YFA0211300), the National Natural Science Foundation of China (Grant No. 12234010), and the Natural Science Foundation of Jiangsu Province (Grant No. BK20233001).

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