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Magnetic-flux induced persistent currents in quasiperiodic mesoscopic rings

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Under the tight-binding approximation, we study the persistent current (PC) in the magnetic-flux-threaded mesoscopic ring which is constructed according to the Fibonacci-class model. It is shown that the energy spectra form band structures and the subbands present a self-similarity. The PC is determined by the magnetic flux, the site energy, and the Fermi level. It is found that the large PC can be observed if the Fermi level reaches a specific value. Our investigation provides detailed information on the structural influence on the PC and contributes a mechanism to understand the large PC observed in the experiments. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1853897]

I. INTRODUCTION

In the past decades, much attention has been paid to the mesoscopic system, where the classical transport theory is invalid and the quantum interference has to be taken into account. The persistent current (PC) in mesoscopic rings is one of the distinguishing features of quantum interference. Since Büttiker *et al.* first predicted the PC in one-dimensional (1D) metal loops in 1983,[1] there have been a lot of theoretical work on this subject.[2–7] The experimental work on PCs has been carried out since last decade. In the early experiment made by Lévy *et al.*,[8] the PC measured on 10^7 copper rings is in agreement with the theoretical prediction under the diffusive case. However, the experimental observation of Chandrasekhar *et al.*[9] indicated that the current in single Au ring is one or two orders of magnitude larger than the value predicted by the noninteracting theory. Some models have been presented to explain this puzzle.[10] Recently more experimental work came out and brought new challenges to the theoretical work, such as the sign of the PC near zero field,[11] the correlation of the PC with the phase coherence time, etc.[12] Up to now, the problem on the PCs has not been well understood. Generally speaking, the disorder of the system and the electron-electron interaction are two important factors influencing the magnitude of the PC. In this work, we study the persistent current in the magnetic-flux-threaded mesoscopic ring which is constructed according to the Fibonacci-class (FC) model. The FC model is a quasiperiodic structure between periodic and disordered cases. In the FC mesoscopic ring, the sites are arranged according to the substitution rules $B \to B^{n-1}A$ and $A \to B^{n-1}AB$, where A and B are two kinds of sites and n is a repeated number. It is found that the large PC can be obtained theoretically in this quasiperiodic system, which gives a mechanism to understand the large PC observed in the experiments.

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II. THE THEORETICAL MODEL

The FC model is a quasiperiodic structure between periodic and disordered cases.[13] The FC sequence contains two kinds of building blocks A and B, and can be obtained by the substitution rules $B \to B^{n-1}A$ and $A \to B^{n-1}AB$, where n is a repeated number. The FC sequence can also be described as a limit of the generation $S_i^{(n)}$. If we let the initial two generations of the FC sequence as $S_1^{(n)}=B$ and $S_2^{(n)}=B^{n-1}A$, we have the ith generation of the FC sequence as

$$S_i^{(n)} = \{S_{i-1}^{(n)}\}^n + S_{i-2}^{(n)} \; (i \geqslant 3). \tag{1}$$

For example, the FC sequence with $n=2$ has the following initial generations: $S_1^{(2)}=B$, $S_2^{(2)}=BA$, $S_3^{(2)}=\{S_2^{(2)}\}^2+S_1^{(2)}=BABAB$, and $S_4^{(2)}=\{S_3^{(2)}\}^2+S_2^{(2)}=BABABBABABBA$. Define the number of building blocks in $S_i^{(n)}$ as $F_i^{(n)}$, then $F_i^{(n)}$ satisfies

$$F_i^{(n)} = nF_{i-1}^{(n)} + F_{i-2}^{(n)} \; (i \geqslant 3), \tag{2}$$

with $F_1^{(n)}=1$ and $F_2^{(n)}=n$. Obviously the FC model can be simplified to be Fibonacci structure in the case of $n=1$.

Now we consider the electronic behavior in 1D FC mesoscopic ring. The FC mesoscopic ring has two kinds of sites (A,B) and in total N sites are arranged according to the FC sequence. Under the tight-binding approximation, without electron-electron interaction, the Schrödinger equation for a spinless electron in a 1D quasiperiodic mesoscopic ring threaded by magnetic flux Φ can be written as

$$(E-\varepsilon_l)C_l = V_{l,l+1}C_{l+1} + V_{l,l-1}C_{l-1}, \tag{3}$$

where C_l is the amplitude of wave function on the lth site, $V_{l,l\pm1}$ is the nearest hopping integral, and ε_l is the site energy. In this work, we restrict ourselves in the on-site model: $V_{l,l\pm1}$ is set as a constant, i.e., as the energy unit, and the site energy ε_l is taken as $\varepsilon_l=\varepsilon_A$ (or ε_B) if atom A (or B) occupies the site in the ring. Thus, Eq. (3) can be expressed in the matrix form,

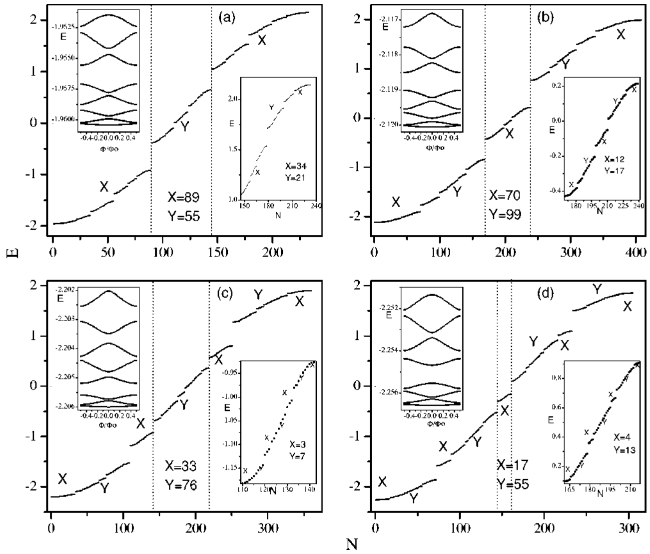

FIG. 1. The energy spectra of the FC rings $S_i^{(n)}$ when $\varepsilon=0.4$ and $\Phi/\Phi_0=0.3$. (a) $S_{12}^{(1)}$—$n=1$, $i=12$, $N=233$; (b) $S_8^{(2)}$—$n=2$, $i=8$, $N=408$; (c) $S_6^{(3)}$—$n=3$, $i=6$, $N=360$; (d) $S_5^{(4)}$—$n=4$, $i=5$, $N=305$. The up insets are the magnetic-flux-dependent energy spectra for $-\frac{1}{2}<\Phi/\Phi_0<\frac{1}{2}$. The down insets are the enlarged subbands to show the self-similarity.

$$\begin{pmatrix} C_{l+1} \\ C_l \end{pmatrix} = M_{l+1,l}\begin{pmatrix} C_l \\ C_{l-1} \end{pmatrix}, \tag{4}$$

where the transfer matrix is

$$M_{l+1,l} = \begin{pmatrix} -(E-\varepsilon_l) & -1 \\ 1 & 0 \end{pmatrix}. \tag{5}$$

Because a magnetic flux Φ threaded through the ring will lead to the twisted boundary condition for the wave function of the electrons, the equation for the global transfer matrix can be written as

$$\begin{pmatrix} C_{N+1} \\ C_N \end{pmatrix} = \overline{M}\begin{pmatrix} C_1 \\ C_0 \end{pmatrix} \equiv e^{i2\pi\Phi/\Phi_0}\begin{pmatrix} C_1 \\ C_0 \end{pmatrix}, \tag{6}$$

where $\overline{M}=\Pi_{l=1}^{N} M_{l+1,l}$ and $\Phi_0=hc/e$ is the flux quantum. By denoting the trace of $\overline{M}$ as $\chi=\frac{1}{2}\,\mathrm{Tr}\,\overline{M}$, the flux-dependent energy $E_n(\Phi)$ can be obtained from

$$\chi = \cos(2\pi\Phi/\Phi_0). \tag{7}$$

The persistent current in the ring contribute by the nth energy level follows:

$$I_n = -c\frac{\partial E_n(\Phi)}{\partial \Phi}, \tag{8}$$

where c is the velocity of the light. At zero temperature, if the number of electrons in the spinless fermion system equals N_e, the total persistent current in the mesoscopic ring satisfies

$$I(\Phi) = \sum_{n=1}^{N_e} I_n(\Phi). \tag{9}$$

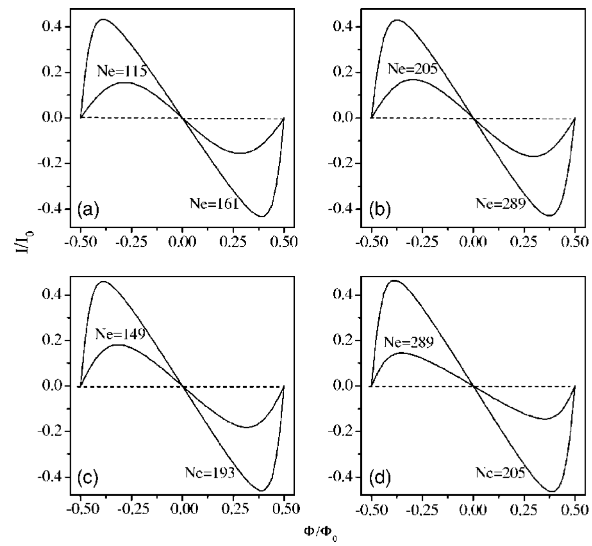

FIG. 2. The persistent currents vs the magnetic flux Φ in the FC rings $S_i^{(n)}$, where $I_0=(4\pi c/N\Phi_0)\sin[(N_e/N)\pi]$ and $\varepsilon=0.4$. (a) $S_{12}^{(1)}$—$n=1$, $i=12$, $N=233$; (b) $S_8^{(2)}$—$n=2$, $i=8$, $N=408$; (c) $S_6^{(3)}$—$n=3$, $i=6$, $N=360$; (d) $S_5^{(4)}$—$n=4$, $i=5$, $N=305$.

III. THE NUMERICAL CALCULATIONS

Based on Eqs. (5)–(9), we can carry out the numerical calculations of the energy spectra and the PC in the FC mesoscopic rings. In the following calculations, we set $\varepsilon_A=-\varepsilon_B=\varepsilon$. In some senses, the parameter ε represents the strength of the quasiperiodicity. Figure 1 shows the energy spectra of the FC rings $S_i^{(n)}$ when $\varepsilon=0.4$ and $n=1,2,3,4$, respectively. The electron eigenenergies form a "band" structure in each FC ring [as shown in the up-insets of Figs. 1(a)–1(d)]. The subbands possess the hierarchical characterization with self-similarity [as shown in Figs. 1(a)–1(d) and therein down-insets]. For example, when $\Phi/\Phi_0=0.3$, there are two types of bands, i.e., "X" and "Y" in the energy spectra of the FC ring with $n=2$. There are five bands "$XYXYX$" in the first hierarchy [as shown in Fig. 1(b)]. In the second hierarchy, each band X will further split into five subbands $XYXYX$ and each Y into three subbands "XYX." Thus, a self-similarity is presented in the case of $n=2$. The self-similarity is also demonstrated in the FC rings with $n=1$, 3, and 4 [as shown in Figs. 1(a), 1(c), and 1(d) and therein down-insets]. Generally speaking, in the FC ring $S_i^{(n)}$, the first hierarchy of the energy spectrum contains $2n+1$ main bands: $n+1$ of type X and n of type Y. The number of levels is $N_X=F_{i-2}^{(n)}$ in the band X and $N_Y=F_{i-1}^{(n)}+F_{i-2}^{(n)}$ in the band Y, respectively. Further in the second hierarchy, previous X splits into $2n+1$ subbands: $n+1$ of type X and n of type Y, where the number of levels is $N_{XX}=F_{i-4}$ in X and $N_{XY}=F_{i-3}-F_{i-4}$ in Y, respectively. Meanwhile, previous Y also further splits into $2n-1$ subbands: n of type X and $n-1$ of type Y, where the number of levels are $N_{YX}=F_{i-3}$ in X and $N_{YY}=F_{i-2}-F_{i-3}$ in Y, respectively. The hierarchy will go on in this way, thus the subbands possess the self-similarity, which is quite similar to the energy spectrum of the FC quasilattices.[13]

On the basis of the energy spectrum, the behavior of the PC in the FC ring can be obtained. Figure 2 gives the flux-dependent PC of the FC rings with two different Fermi lev-

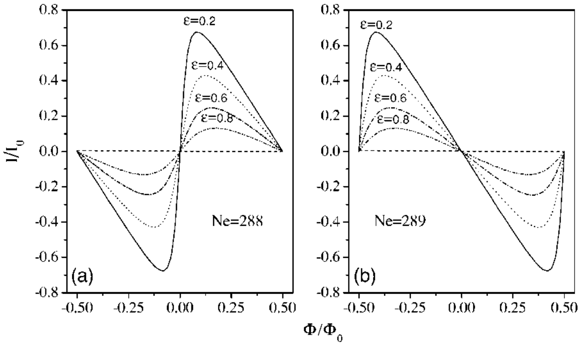

FIG. 3. The persistent currents vs the magnetic flux Φ in the FC ring $S_8^{(2)}$ with even and odd electron-filling number N_e under different site energies ε, where the total number of sites is $N=408$. (a) $N_e=288$ and (b) $N_e=289$.

els, where $\varepsilon=0.4$. First, it is shown that the PC shows a sinusoidallike relation respect to the magnetic flux. And the PC depends on the Fermi level or the electron-filling number. In the second, the PC is also determined by the site energy. Figure 3 plots the persistent currents versus the magnetic flux Φ in the FC ring $S_8^{(2)}$ with even and odd electron-filling number N_e under different site energies ε. It is found that if ε increases, the PC is gradually suppressed. Increasing ε means that the quasiperiodicity in the structure becomes stronger, then the dependence of the energy level on the flux becomes smoother. Therefore the current contribution coming from these energy levels will decrease according to Eq. (8). Consequently, the total current will decrease. In the third, at the zero-temperature limit, the sign of the PC is determined by the electron filling number N_e. In the ring with an even number of electrons, the flux-dependent PC [as shown in Fig. 3(a)] is like that for a paramagnet, whereas in a ring with an odd number of electrons, the flux-dependent PC [as shown in Fig. 3(b)] behaves like that in a diamagnet. This property is also called the parity effect.

Now we are interested in how to observe the large PC in the mesoscopic FC rings. Figure 4 plots the PC in the FC ring versus the odd number of filling electrons N_e when $\Phi/\Phi_0=-0.4$ and $\varepsilon=0.4$. Obviously when N_e comes to some special values, the PC in the quasiperiodic FC ring can be close to the periodic case. In details, I/I_0 can be more than 0.4 in the FC ring if the Fermi energy takes some peculiar values. Here $I_0=(4\pi c/N\Phi_0)\sin[(N_e/N)\pi]$ is the maximum of PCs in a periodic structure). In these cases, the PC is absent of suppression, therefore, large PC can be observed in the FC rings. In the experiment, one may intentionally introduce the nodes in the ring to realize the FC mesoscopic model. By changing the magnetic flux and the electron number in the ring, the large PC may be obtained.

IV. SUMMARY

In this work, we study the persistent current in the magnetic-flux-threaded mesoscopic ring which is constructed according to the FC model. Under the tight-binding approximation, the flux-dependent energy spectra and the PC are theoretically obtained by applying transfer-matrix technique. It is shown that the energy spectra form band structures and the subbands present a self-similarity. The PC is determined by the magnetic flux, the site energy and the Fermi level. The

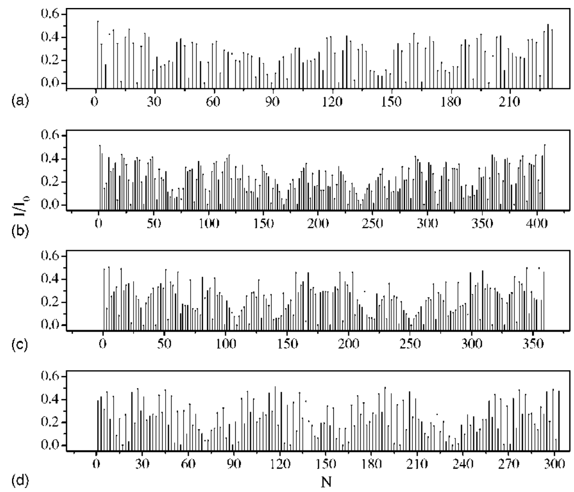

FIG. 4. The persistent current vs the odd number of filling electrons N_e in the FC rings $S_i^{(n)}$, where $I_0=(4\pi c/N\Phi_0)\sin[(N_e/N)\pi]$, $\Phi/\Phi_0=-0.4$ and $\varepsilon=0.4$. (a) $S_{12}^{(1)}$—$n=1$, $i=12$, $N=233$; (b) $S_8^{(2)}$—$n=2$, $i=8$, $N=408$; (c) $S_6^{(3)}$—$n=3$, $i=6$, $N=360$; (d) $S_5^{(4)}$—$n=4$, $i=5$, $N=305$.

PC shows a sinusoidallike relation respect to the magnetic flux, and the increment of the site energy leads to a dramatic suppression of the PC. Further, it is found that if the Fermi level goes to a specific value, the PC in the FC ring has absence of suppression, i.e., the large PC can be observed in this case. The parity effect in the FC systems is also discussed. Our investigation provides detailed information on the structural influence on the PC and contributes a mechanism to understand the large PC observed in the experiments.

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